EXAM II, MA3042, Some quarter   Name:

This is not precisely an old exam, but all of these problems have appeared on past exams. Problems 1–4 would make a good exam, as would problems 1–3 together with 5.

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1. Let $S = \text{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\}$. Find a basis for $S^\perp$.

2. Suppose that we have experimental data as shown in the accompanying table.

x	1	2	3	4
y	1	2	2	3

   (a) Formulate the matrix equation $A\mathbf{x} = b$ that results from attempting to fit a line to the given data, and show that $\mathbf{b} \notin CS(A)$.

   (b) Factor A as $A = QR$, and use this factorization to obtain a least-squares solution $\hat{\mathbf{x}}$ to the equation from (a).

   (c) Calculate the residual $r(\hat{\mathbf{x}}) = \mathbf{b} - A\hat{\mathbf{x}}$.

   (d) Verify that $r(\hat{\mathbf{x}}) \in N(A^T)$

3. Let $\theta \in \mathbf{R}$, $\mathbf{x}_1 = (\cos\theta, \sin\theta)^T$, and $\mathbf{x}_2 = (-\sin\theta, \cos\theta)^T$.

   (a) Show that $\{\mathbf{x}_1, \mathbf{x}_2\}$ is an orthonormal basis for $\mathbf{R}^2$.

   (b) Given a vector $\mathbf{y} = (y_1, y_2)^T \in \mathbf{R}^2$, write $\mathbf{y}$ as a linear combination $c_1\mathbf{x}_1 + c_2\mathbf{x}_2$.

   (c) Verify that $c_1^2 + c_2^2 = ||\mathbf{y}||^2 = y_1^2 + y_2^2$.

4. Let $A = \begin{bmatrix} a & 1 & -2 \\ 0 & 1 & 4 \\ 0 & 4 & 1 \end{bmatrix}$. For precisely what values of a is A diagonalizable? Explain.

5. Let $A = \begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix}$. Find a diagonalizing matrix X for A, and calculate $\lim_{n \to \infty} A^n$.